Some Recent Research in Mathematics Education

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The speaker will give an overview of recent research conducted by Ball, Ma, Stigler, Hiebert, Stevenson, Carpenter, and other investigators. He will survey what we have learned from cross-cultural studies such as TIMSS and the work of Liping Ma. The concept of Japanese-style "lesson study" (a collaborative form of action research) will be discussed in detail. The speaker (with Chris Nugent) is co-director of an Iowa Eisenhower grant project to pilot Japanese-style lesson study in Dubuque.

RECOMMENDED BOOKS

A complete set of slides from this talk is available at our Web site.
Links and additional references can also be found there.

Iowa Council of Teachers of Mathematics
Annual Meeting
February 22, 2002
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I hear and I forget,
I see and I remember,
I do and I understand.

Confucius
Examples (Carpenter et al)

1. Fill in the blank: \[ 4 + 8 = [ \ ] + 5 \]
   - 6th-graders in Madison: 0 out of 32 correct.
   - "Above average" 6th-graders in Dubuque: 1 out of 6 correct.
   - Precalculus students at Loras: 23 out of 23 correct.

2. Give an example of a number between 3.1 and 3.11:
   - "Above average" 6th-graders in Dubuque: 0 out of 6 correct.
   - Precalculus students at Loras: 17 out of 23 correct.
A Nation at Risk

- Full title: "A Nation at Risk: The Imperative for Educational Reform"
- Published April 1983
- Authors: 18-member "National Commission on Excellence in Education"
- Report to U.S. Secretary of Education (and to the nation)
The educational foundations of our society are presently being eroded by a rising tide of mediocrity that threatens our very future as a nation …

If an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as an act of war …

We have, in effect, been committing an act of unthinking, unilateral educational disarmament …

Our society and its educational institutions seem to have lost sight of the basic purposes of schooling, and of the high expectations and disciplined effort needed to attain them …

International comparisons of student achievement, completed a decade ago, reveal that on 19 academic tests American students were never first or second and, in comparison with (students of) other industrialized nations, were last seven times …

Some 23 million American adults are functionally illiterate ….
from *A Nation at Risk*

- Functional illiteracy among minority youth may run as high as 40 percent …

- The College Board's Scholastic Aptitude Tests (SAT) demonstrate a virtually unbroken decline from 1963 to 1980. Average verbal scores fell over 50 points and average mathematics scores dropped nearly 40 points …

- Between 1975 and 1980, remedial mathematics courses in public four-year colleges increased by 72 percent and now constitute one-quarter of all mathematics courses taught in those institutions …

- Business and military leaders complain that they are required to spend millions of dollars on costly remedial education and training programs …

- Excellence characterizes a school or college that sets high expectations and goals for all learners, then tries in every way possible to help students reach them …

- We find that for too many people education means doing the minimum work necessary for the moment, then coasting through life on what may have been learned in its first quarter …

- In some metropolitan areas, basic literacy has become the goal rather than the starting point …
from *A Nation at Risk*

- In some colleges, maintaining enrollments is of greater day-to-day concern than maintaining rigorous academic standards …

- Secondary-school curricula have been homogenized, diluted and diffused to the point that they no longer have a central purpose. In effect, we have a cafeteria-style curriculum in which the appetizers and desserts can easily be mistaken for the main courses …

- The amount of homework for high-school seniors has decreased (two-thirds report less than one hour a night) and grades have risen as average student achievement has been declining …

- Principals and superintendents must play a crucial leadership role in developing school and community support for the reforms we propose …

- Excellence costs. But in the long run, mediocrity costs far more …
from *A Nation at Risk*

Some concerns about teacher preparation:

- Too many academically weak students in teacher preparation programs.
- Too many educational methods courses, and not enough content preparation.
- Low salaries.
- Severe shortages in some areas, especially in math, science and special education.
- Too many teachers teaching "out of field".
from *A Nation at Risk*

Some ideas to improve teacher preparation:

- High educational standards for preservice teachers.
- Higher salaries, with an effective system of evaluation and rewards.
- More opportunities for professional development.
- Alternate certification programs to address the shortage of math and science teachers.
- Incentives such as grants and loans to attract outstanding students to the teaching profession.
- Master teachers should help design teacher-preparation programs and help supervise beginning teachers.
TIMSS

Fourth-Grade Achievement (Mathematics)

❖ Nations with average scores significantly higher than that of the U.S.:

  Singapore
  Korea
  Japan
  Hong Kong

❖ Nations with average scores not significantly different from that of the U.S.:

  Australia
  United States
  Canada

❖ Nations with average scores significantly below that of the U.S.:

  England
  Norway
  Greece
  Portugal
TIMSS

Eighth-Grade Achievement (Mathematics)

- Nations with average scores significantly higher than that of the U.S.:
  - Singapore
  - Korea
  - Japan
  - Hong Kong
  - France
  - Russia
  - Canada

- Nations with average scores not significantly different from that of the U.S.:
  - Germany
  - England
  - Norway
  - United States
  - Greece

- Nations with average scores significantly below that of the U.S.:
  - Portugal
  - Iran
Development of Student Achievement (TIMSS)
## NAEP (1996): Mathematics

<table>
<thead>
<tr>
<th>Grade</th>
<th>Below Basic</th>
<th>Basic</th>
<th>Proficient</th>
<th>Advanced</th>
</tr>
</thead>
<tbody>
<tr>
<td>4th</td>
<td>36%</td>
<td>43%</td>
<td>19%</td>
<td>2%</td>
</tr>
<tr>
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<td>38%</td>
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<td>20%</td>
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</tr>
<tr>
<td>12th</td>
<td>31%</td>
<td>53%</td>
<td>14%</td>
<td>2%</td>
</tr>
</tbody>
</table>
Japan versus U.S.: Miscellaneous Cultural Factors

- National curriculum.
- High stakes tests for high school students.
- Parental expectations.
- Parent literacy and academic interests.
- Cultural expectations.
- Math is considered to be intrinsically interesting.
- Academic interests of students.
- Limited use of technology in the early grades.
- Inschool professional development of teachers.
- Coherent lessons.
- No interruptions.
- Focussed curriculum (versus U.S.: "mile wide and inch deep")
- Stranded curriculum (versus U.S.: "spiral" going around and around but never getting higher).
- Simpler number names.
- Simpler measurement system.
- Cram schools ("juku").
Japan versus U.S.: Miscellaneous Cultural Factors

- Respect for elders.
- Disciplined, orderly behavior.
- Sense of community.
- Mastery of the "basics". Example: all Japanese 2nd graders learn "ku-ku" (multiplication through 10×10) by heart in second grade.
- Longer school year.
- Higher compensation for teachers.
- Hard work.
- Confucianism.
Japan versus U.S.: Non-factors

- Homogeneous society (cf. Singapore).
- Tracking.
- Class size.
- Amount of television.
- Preservice teachers coursework, degrees, etc.
- Time spent on homework (?)
- Time spent in school (?)
Asian Classroom

- confident students
- working hard
- having fun
- challenging problems
- excited about learning
- sharing learning
- working on problems:
  - individually, then
  - in groups, then
  - selected students present solutions on the board.
Math Education in Asia

- Mathematics …
  - is a collection of related ideas that make sense;
  - is not a set of meaningless procedures to be memorized.

- Curriculum and textbooks are focused on the essentials.

- A limited number of terms are introduced in each lesson.

- Any terms or concepts that are introduced are heavily used and thoroughly mastered in problems, applications, and proofs.
Looking at the situation as a whole, one might even argue that Japanese lessons better exemplify current U.S. reform ideas than do U.S. lessons …

Japanese lessons, for example, emphasized student thinking and problem solving, multiple solution methods, and the kinds of discourse described in U.S. reform documents to a greater extent than U.S. lessons did …

And this is not the worst of it. When we examined the places in the video that teachers referred to as examples of reform, we saw … that reform teaching, as interpreted by some teachers, might actually be worse than what they were doing previously in their classrooms …

The United States is always reforming but not always improving …

The most alarming aspect of classroom teaching in the United States is not how we are teaching now but that we have no mechanism for getting better …
Lesson Study in Japan

Step 1: Define the problem.
Step 2: Plan a lesson.
Step 3: Teach the lesson.
Step 4: Group critique of the lesson.
Step 5: Revise the lesson.
Step 6: Teach the revised lesson.
Step 7: Evaluate and reflect again.
Step 8: Share the results.
Participation in school-based professional development groups is considered to be part of the teacher’s job in Japan …

In lesson study, groups of teachers meet regularly over long periods of time … to work on the design, implementation, testing, and improvement of one or several ‘research lessons’…

By all indications, lesson study is extremely popular and highly valued by Japanese teachers, especially at the elementary school level. It is the linchpin of the improvement process …

The focus is on the lesson, not on the teacher who taught the lesson; the lesson, after all, is a group product, and all members of the group feel responsible for the outcome of their plan.

Teachers in Japan see themselves as developing the profession as well as themselves. Few U.S. teachers would feel this way…Japan has succeeded in developing a system that not only develops teachers but also develops knowledge about teaching that is relevant to classrooms and sharable among the members of the teaching profession.
from *The Teaching Gap*

The lesson-study process has an *unrelenting focus on student learning*. All efforts to improve lessons are evaluated with respect to *clearly specified learning goals*, and revisions are always justified with respect to *student thinking and learning*… Although this feature *might seem obvious and trivial, it is not*. Reforms in the United States are often tied to particular *theories of teaching* or to *educational fads* instead of *specific learning outcomes*. Because of this, success often is measured by the degree to which teachers implement recommended practices. Someone is marked as a good teacher because he or she uses *cooperative groups* or *concrete manipulatives*, instead of on the basis of his or her students’ *successful learning* …
Liping Ma's *Knowing and Teaching Elementary Mathematics*

- Chinese teachers possess a “profound understanding of fundamental mathematics” that far exceeds that of the best teachers in this country …

- Chinese teachers “understanding of the mathematics they teach and—equally important—of the ways that elementary mathematics can be presented to students continues to grow throughout their professional lives …”

- Chinese teachers have a much deeper understanding of “pedagogical content knowledge”

- How they attained this knowledge: “studying teaching materials intensively …”

- “Chinese teachers not only study teaching materials individually, they also do it with their colleagues … Chinese teachers are organized in jiaoyanzu or 'teaching research groups' … These groups, usually meeting once a week for about one hour, get together formally to share their ideas and reflections on teaching…”
Liping Ma's *Knowing and Teaching Elementary Mathematics*

- In China, “a teacher’s subject matter knowledge of mathematics, which develops under a concern of teaching and learning, will be relevant to teaching and is likely to be used in teaching. In other words, the Chinese teachers develop and deepen their subject matter knowledge of elementary mathematics by preparing for classes, teaching the material, and reflecting on the process. Therefore, what they learn will contribute to and be used in teaching …”

- In the United States “… it seems that low-quality school mathematics education and low-quality teacher knowledge of school mathematics reinforce each other. Teachers who do not acquire mathematical competence during schooling are unlikely to have another opportunity to acquire it …”
Liping Ma's *Knowing and Teaching Elementary Mathematics*

“Time is an issue here. If teachers have to find out what to teach by themselves in their very limited time outside the classroom and decide how to teach it, then where is the time for them to study carefully what they are to teach? U.S. teachers have less working time outside the classroom than Chinese teachers … but they need to do much more in this limited time. What U.S. teachers are expected to accomplish, then, is impossible …”
Development of Teacher Knowledge in US and China

![Graph showing the development of teacher knowledge in the United States and China over teaching experience. The graph indicates that teacher knowledge increases with teaching experience in China, while it decreases in the United States.]
NCTM's Six Principles

1. Equity
2. Curriculum
3. Teaching
4. Learning
5. Assessment
6. Technology
NCTM's Five Content Standards

1. Number and Operations
2. Geometry
3. Measurement
4. Algebra
5. Data Analysis and Probability
NCTM's Five Process Standards

1. Problem Solving
2. Reasoning and Proof
3. Communication
4. Connections
5. Representation
Glenn Commission Report

Before It's Too Late
(September 2000)

- Goal #1: A *system* to improve K-12 teaching of Math and Science.
- Goal #2: Increase quantity and quality of Math and Science teachers.
- Goal #3: Improve the environment of Math and Science teachers.
- "Needs assessment" for professional development.
- "Inquiry groups" (lesson study groups).
- Summer institutes.
- Recommended federal spending for the *first year*:
  - $15 million for needs assessment;
  - $1.2 billion for summer institutes;
  - $1.6 billion for inquiry groups;
  - $112 million for leadership training;
  - $120 million for Academies/Fellows;
  - $3 billion total.
From Chapter Two (General Recommendations):

- Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach.

- Prospective elementary grade teachers should be required to take at least 9 semester-hours on fundamental ideas of elementary school mathematics.

- Courses on fundamental ideas of school mathematics should focus on a thorough development of basic mathematical ideas.

- More mathematicians should consider becoming deeply involved in K-12 mathematics education.
Mathematical Education of Teachers (CBMS, 2001)

From Chapter Two (General Recommendations):

- Teachers need the opportunity to develop their understanding of mathematics and its teaching throughout their careers, through both self-directed and collegial study, and through formal coursework. In some countries where student achievement is high, teachers, alone and in groups, spend time refining their lessons and studying the underlying mathematics. They observe each others classes … More professional development opportunities of this kind are essential for U.S. teachers.

- Mathematics in middle grades (grades 5-8) should be taught by mathematics specialists.
Lesson Study Projects

- Columbia University Lesson Study Research Group
- Bok Center at Harvard
- Delaware Department of Education
- PR-SSI (Puerto Rico Statewide Systemic Initiative)
- Paterson (NJ) School #2
- Bellevue (WA) School District
- NRC (National Research Council)
- Mills College (CA)
- School Renewal Web Center
- Mathematics Teaching and Learning to Teach Project (University of Michigan)
Teaching and Learning Mathematics: A Problem-Solving Approach

- Funding provided by the Board of Regents of the State of Iowa through a Dwight D. Eisenhower Professional Development Grant.

- Additional funding provided by Loras College, Dubuque Community Schools, and Dubuque Holy Family Catholic Schools.

- Summer 2002: Institute on math ed (research, standards, and content) for 10 teams of elementary school teachers (public and parochial) with 3 teachers on a team.

- 2002-2003 school year: Participating teams will pilot a Japanese-style lesson study process 90 minutes a week in their school.

- February 2003: Team presentation at the next ICTM meeting.

- May 2003: Lesson Study Fair on the campus of Loras College.

- We are currently exploring funding opportunities for followup projects (Title IIA? NSF?).
Assessment

- One of NCTM's Six "Principles".
- Philosophy: Karl Popper, W. E. Deming, Tom Peters.
- Standards: NCTM, State, District.
- Peter Holly and Mary Daily Lange.
- Data Coach Training.
- School Improvement Plans.
- ITBS.
- Sandy Moore.
- Multiple means of assessment.
Follow-up Projects

- ESEA (Elementary and Secondary Education Act) recently passed by Congress and signed by President Bush: $26 billion.

- Eisenhower program (as such) eliminated.

- Title IIA: "Teacher and Principal Quality Training," consolidates class size reduction and Eisenhower funds: $22 million for Iowa next year.

- Title IIB: "Math and Science Partnership" program at DoED: originally slated to receive $450 million this year, but only $12.5 million was appropriated!

- NSF "Math and Science Partnership" program: $160 million this year; $1 billion over five years.

- Primary criteria for new programs: helping disadvantaged students, e.g., Title I schools.

- A model program: PR-SSI.

- Iowa assets: UNI, AEAs, ICN.

- Blend inservice and preservice programs?