

A Tale of Nine Points

Angela Kohlhaas

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This was largely a chalkboard talk, but I used slides for the historical personalities.

References

- David Eisenbud, Mark Green, and Joe Harris, "Cayley-Bacharach Theorems and Conjectures," *Bulletin of the American Mathematical Society*, **33** (3), 1996.
- John Stillwell, *Yearning for the Impossible: The Surprising Truths of Mathematics*, A K Peters, Ltd., 2006.
- Wikipedia, mathematician pictures, <http://en.wikipedia.org>, accessed November 2009.

Pappus of Alexandria

- Circa 320 AD
- Wrote the eight-book *Collection*
- Proposition 139 Book VII
- Pappus' Theorem: Nine points and nine lines
- Need "line at infinity" for theorem to hold in all cases
- 1600s: Kepler and conic sections as planetary orbits

Girard Desargues



- 1639: Sketch of What Happens When a Cone Meets a Plane
 - Formalizes Projective Plane
 - Abraham Bosse: artist using perspective
 - Étienne Pascal: son Blaise
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- Lone copy remains after 200 years (Fermat and Descartes' coordinate geometry prevails in the meantime)

Blaise Pascal

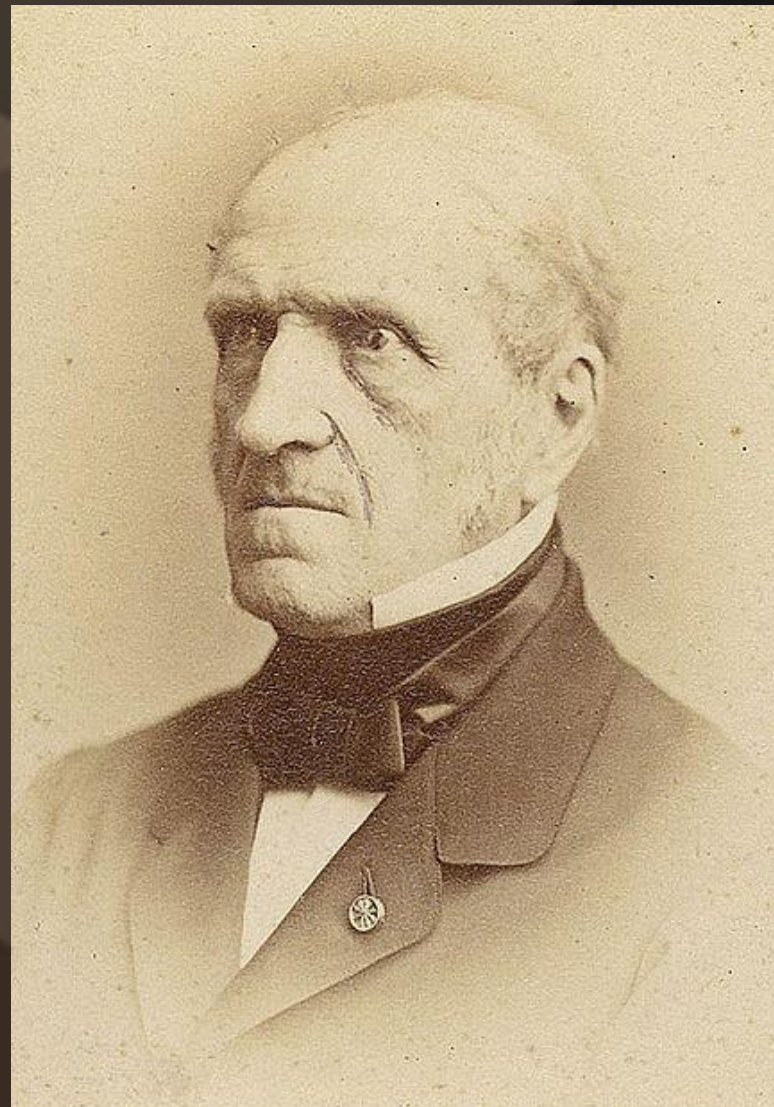
- 1640: Reads Desargues' paper at age 16 and proves



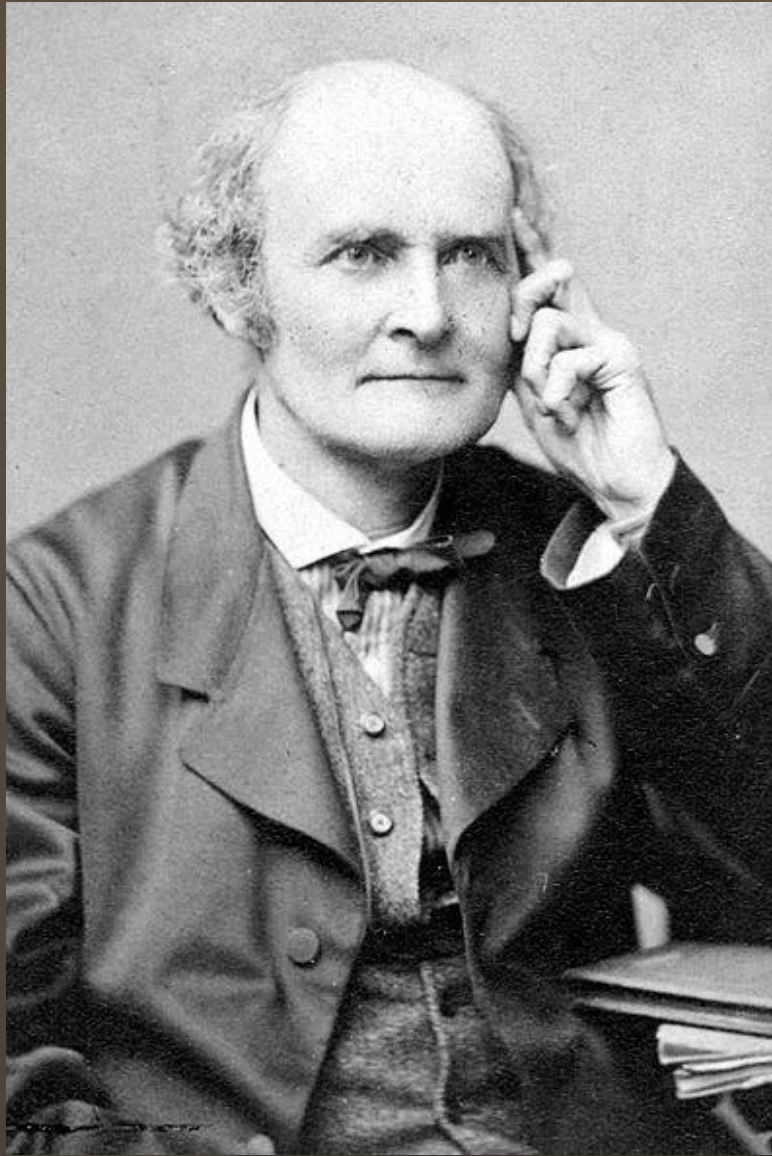
- Pascal's Mystical Hexagon Theorem:
 - Nine points, seven lines, and a conic
 - Uses his theorem to prove many results about conics
 - Tangent lines

Michel Chasles

- 1837: *Historical view of the origin and development of methods in geometry*
- Chasles' Theorem:
- Nine points and three cubics
- Great math, poor historical common sense



Arthur Cayley



- 1843: Generalizes Chasles' Theorem...
- But incorrectly!
- Higher degree curves also produce sets of points imposing independent conditions
- 1881: I. Bacharach fixes Cayley's statement, giving us the Cayley-Bacharach Theorem

20th Century

Geometry

Schemes

- Sets of points
- Lines and curves
- surfaces

Algebra

Rings

- Rings R/I
- R = ring of all polynomials
- I = ideal of some polynomials

Theorem: Polynomial rings over fields are Gorenstein.